

Revisiting the Thermal Stability of Radiation-dominated Thin Disks

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ABSTRACT

The standard thin disk model predicts that when the accretion rate is over a small fraction of the Eddington rate, which corresponds to $L \gtrsim 0.06L_{\text{Edd}}$, the inner region of the disk is radiation-pressure-dominated and thermally unstable. However, observations of the high/soft state of black hole X-ray binaries with luminosity well within this regime ($0.01L_{\text{Edd}} \lesssim L \lesssim 0.5L_{\text{Edd}}$) indicate that the disk has very little variability, i.e., quite stable. Recent radiation magnetohydrodynamic simulations of a vertically stratified shearing box have confirmed the absence of the thermal instability. In this paper, we revisit the thermal stability by linear analysis, taking into account the role of magnetic field in the accretion flow. By assuming that the field responses negatively to a positive temperature perturbation, we find that the threshold of accretion rate above which the disk becomes thermally unstable increases significantly compared with the case of not considering the role of magnetic field. This accounts for the stability of the observed sources with high luminosities. Our model also presents a possible explanation as to why only GRS 1915+105 seems to show thermally unstable behavior. This peculiar source holds the highest accretion rate (or luminosity) among the known high state sources, which is well above the accretion rate threshold of the instability.

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1. Introduction

The standard thin disk is a milestone in the development of accretion disk theory. It successfully explains many observations such as the “big blue bump” in active galactic nuclei (see review in Frank et al. 2002) and the spectrum of high/soft state of black hole X-ray binaries (Gierliński & Done 2004; hereafter GD04). However, some questions still remain unsolved (e.g., Koratkar & Blaes 1999). One of them, for example, is how to explain the observed hard X-ray emission in black hole sources. The temperature of the thin disk is too low to produce the X-ray emission.

In this paper we will discuss another puzzle, which is its thermal stability. Since the discovery of the standard thin accretion disk (e.g., Shakura and Sunyaev 1973), many efforts have been made to examine its various stability. When the mass accretion rate is higher than a few percent of Eddington rate ($\dot{M}_{\text{Edd}} \equiv 10L_{\text{Edd}}/c^2$), which roughly corresponds to $\gtrsim 0.06L_{\text{Edd}}$, the innermost region of the disk will be radiation-pressure-dominated. It was found that in this case the disk will be both secularly (Lightman & Eardley 1974) and thermally (Shakura & Sunyaev 1976; Piran 1978) unstable. Time-dependent global numerical calculations (Honma et al. 1992; Szuszkiewicz & Miller 1998; Janiuk et al. 2002; Li et al. 2007) found that the local thermal instability will result in the “limit-cycle” behavior. However, observations of the high/soft state of black hole X-ray binaries have raised doubt to the above prediction (GD04). It has been well established that the accretion flow in this state is described by the standard thin disk model (Zdziarski & Gierliński 2004; Done et al. 2007). The luminosity of the sources compiled in GD04 ranges from 0.01 to 0.5 L_{Edd} , with the highest one even exceeding the theoretical stable limit by nearly one order of magnitude. Thus we should expect to see the limit-cycle behavior. In contrast to this expectation, however, observations show little variability which convincingly indicates that they are thermally stable.

Some efforts have been made to solve this puzzle. For example, if a large fraction of the dissipated energy is channeled into a corona or outflow/jet the disk would be stable (Svensson & Zdziarski 1994). However, the sources adopted in GD04 are all disk-dominated. The absence of hard X-ray and radio emission directly rules out the existence of corona or jet. Another idea is to assume that the viscous stress is proportional to the gas pressure only, rather than the sum of the gas and radiation pressure (Lightman & Eardley 1974; Stella & Rosner 1984). As pointed out in GD04, the problem with this idea is that the outcome of this modified viscosity is in conflict with the observation that the color temperature correction is constant for different accretion rates. Moreover, on the theoretical side, in contrast to the above two ideas, numerical simulations have shown that only a very small fraction of the dissipated energy is channeled into the corona, which is consistent with the absence of hard

X-ray emission in high state of black hole X-ray binaries, and the stress is proportional to the sum of gas and radiation pressure (Hirose et al. 2006; Hirose et al. 2009a). Recently, the issue of thermal instability has been addressed by the radiation magnetohydrodynamic simulations of a vertically stratified shearing box (Hirose et al. 2009a). Their results indicate that the radiation-dominated disk is stable over ~ 40 cooling timescales. They explained the stability as the lack of correlation between the stress and the pressure within the cooling timescale.

In this paper, we revisit the thermal stability of thin disks by linear analysis. Different from previous analyses, we include the role of the magnetic pressure and further assume that the magnetic field becomes weaker when the gas temperature increases. Our analysis indicates that the onset accretion rate for the instability becomes larger compared with the previous result. The organization of our paper is as follows. In §2, we deliver the equations and conduct an analysis of the thermal instability. The results are shown in §3. The last section (§4) is devoted to a summary and discussion.

2. Accretion disk model with toroidal magnetic fields

2.1. Basic equations

The disk structure is described by the following equations. The equation of vertical hydrostatic equilibrium is

$$\frac{\partial p}{\partial z} + \rho \frac{\partial \psi}{\partial z} = 0, \quad (1)$$

where p is the total pressure varying with z , ρ is the mass density, ψ is the gravitational potential which is assumed to be of the Newtonian form, i.e., $\psi(R, z) = -GM/\sqrt{R^2 + z^2}$, where M is the mass of the black hole. Under the thin disk approximation, we have $\partial p/\partial z \simeq -P_{\text{tot}}/H$ and $\partial \psi/\partial z \simeq GMH/R^3$, where H is the scale height, and P_{tot} is the total pressure at the midplane. Defining the surface density as $\Sigma \equiv 2\rho H$, we have

$$P_{\text{tot}} = P_{\text{gas}} + P_{\text{rad}} + P_{\text{mag}} = \frac{GM\Sigma H}{2R^3}. \quad (2)$$

Here the gas pressure $P_{\text{gas}} = 2k_B\rho T/m_H$, where k_B is the Boltzmann constant, m_H is the hydrogen mass, and T is the temperature at the midplane. The radiation pressure $P_{\text{rad}} = aT^4/3$, where a is the radiation constant. Since in the innermost region of the accretion flow the magnetic field is dominated by its azimuthal component B_φ , the magnetic pressure $P_{\text{mag}} = B_\varphi^2/8\pi$.

The energy equation is written as the balance between the viscous heating and the

cooling through radiation and advection, i.e.,

$$Q_{\text{vis}}^+ = Q_{\text{rad}}^- + Q_{\text{adv}}^- \quad (3)$$

The vertically integrated viscous heating rate is

$$Q_{\text{vis}}^+ = -T_{r\varphi} R \frac{d\Omega}{dR}, \quad (4)$$

where $T_{r\varphi}$ is the stress (see eq. 8 below), and $\Omega = \Omega_K$ with $\Omega_K \equiv \sqrt{GM/R^3}$ is the Keplerian angular velocity. The radiative cooling rate is

$$Q_{\text{rad}}^- = \frac{32\sigma T^4}{3\tau}, \quad (5)$$

where $\tau = \kappa\Sigma/2$ is the vertical optical depth, $\kappa \simeq 0.4 \text{ cm}^2 \text{ g}^{-1}$, since the opacity is dominated by the electron scattering in the inner region of the disk, and σ is the Stefan-Boltzmann constant. The advective cooling term is given by (Abramowicz et al. 1995)

$$Q_{\text{adv}}^- = \frac{\xi \dot{M} \Omega_K^2 H^2}{2\pi R^2}. \quad (6)$$

We set $\xi = 1.5$ in the present work. Our calculations indicate that the role of advection term is negligible in term of the stability analysis of the radiation-dominated thin disk. This term is included for the discussion of the thermal equilibrium solutions in §3.2, which contain optically thick advection-dominated accretion disks (or slim disks; Abramowicz et al. 1988).

The angular momentum conservation equation is expressed as

$$\dot{M}(\Omega_K R^2 - l_{\text{in}}) = 2\pi R^2 T_{r\varphi}, \quad (7)$$

where $l_{\text{in}} = \sqrt{GM R_{\text{in}}}$ is the specific angular momentum at the inner edge of the disk, and R_{in} is fixed to be $3R_g$, with $R_g \equiv 2GM/c^2$ being the Schwarzschild radius.

It is now widely believed that the MHD turbulence associated with the magneto-rotational instability (MRI) is the main mechanism for the angular momentum transfer in accretion disks (e.g., Balbus & Hawley 1998). MHD simulations have shown that, the stress accounting for the angular momentum transport is dominated by the Maxwell stress, rather than the Reynolds stress (Hawley et al. 1995). Besides, we have also learnt from the simulations that the ratio of the Maxwell stress to the total pressure nearly maintains constant (Hawley & Krolik 2001; Machida et al. 2006; Pessah et al. 2007; Hirose et al. 2009a). We therefore have

$$T_{r\varphi} = 2\alpha P_{\text{tot}} H, \quad (8)$$

where α is the viscous parameter.

To close the set of equations, we need one more constraint, which is to connect the magnetic field with other physical quantities. As the thermal instability is mainly concerned in this study, here, we relate the response of the magnetic field to the perturbation of the scale height in a linearized fashion, i.e.,

$$\frac{\delta B_\varphi}{B_\varphi} = -\gamma \frac{\delta H}{H}, \quad (9)$$

where γ is a free parameter. The integrated form of eq. (9) is $\Phi_\gamma \equiv B_\varphi H^\gamma = \text{constant}$. We assume that $\gamma > 0$, i.e., the magnetic field will become weaker with an increase of height (or temperature). This assumption is supported by the MHD numerical simulation of Machida et al. (2006) where they found that the magnetic field becomes stronger when the disk shrinks vertically, although we note that their simulation is for a hot accretion flow rather than a thin disk. We will discuss the physical meaning of Φ_γ in §4. With the above equations we can obtain a thermal equilibrium solution at a certain radius for given M , \dot{M} , α and Φ_γ (§3.2).

2.2. Thermal instability analysis

Since the thermal instability timescale is much shorter than the viscous timescale, the surface density Σ is taken to be constant. Defining $\beta_{\text{gas}} \equiv P_{\text{gas}}/P_{\text{tot}}$ and $\beta_{\text{mag}} \equiv P_{\text{mag}}/P_{\text{tot}}$, we can obtain from eqs. (2), (4)-(6) that

$$\begin{aligned} d \ln P_{\text{tot}} &= d \ln H \\ &= \beta_{\text{gas}}(d \ln T - d \ln H) + 4(1 - \beta_{\text{gas}} - \beta_{\text{mag}})d \ln T + 2\beta_{\text{mag}}d \ln B_\varphi, \end{aligned} \quad (10)$$

and

$$d \ln Q_{\text{vis}}^+ - d \ln(Q_{\text{rad}}^- + Q_{\text{adv}}^-) = d \ln T_{r\varphi} - 4(1 - f_{\text{adv}})d \ln T + f_{\text{adv}}(d \ln \dot{M} + 2d \ln H), \quad (11)$$

where the advection factor f_{adv} is defined as $f_{\text{adv}} \equiv Q_{\text{adv}}^-/(Q_{\text{rad}}^- + Q_{\text{adv}}^-)$. Eqs. (7)-(9) give that

$$d \ln \dot{M} = d \ln T_{r\varphi} = d \ln P_{\text{tot}} + d \ln H, \quad (12)$$

and

$$d \ln B_\varphi = -\gamma d \ln H. \quad (13)$$

Combining eqs. (10) and (13), we have

$$d \ln P_{\text{tot}} = d \ln H = \frac{4 - 3\beta_{\text{gas}} - 4\beta_{\text{mag}}}{1 + \beta_{\text{gas}} + 2\gamma\beta_{\text{mag}}} d \ln T. \quad (14)$$

Substituting eqs. (3), (12) and (14) into eq. (11), we finally get

$$\left[\frac{\partial(Q_{\text{vis}}^+ - Q_{\text{rad}}^- - Q_{\text{adv}}^-)}{\partial T} \right]_{\Sigma} \frac{T}{Q_{\text{vis}}^+} = \frac{2 - 5\beta_{\text{gas}} - 4(1 + \gamma)\beta_{\text{mag}} - 6f_{\text{adv}} + 8f_{\text{adv}}\beta_{\text{gas}} + (8 + 4\gamma)f_{\text{adv}}\beta_{\text{mag}}}{1 + \beta_{\text{gas}} + 2\gamma\beta_{\text{mag}}}. \quad (15)$$

Since the thermal instability condition is $[\partial(Q_{\text{vis}}^+ - Q_{\text{rad}}^- - Q_{\text{adv}}^-)/\partial T]_{\Sigma} > 0$, and the denominator of eq. (15) is always positive when $\gamma > 0$, then the thermal instability criterion can be taken as

$$\Delta = 2 - 5\beta_{\text{gas}} - 4(1 + \gamma)\beta_{\text{mag}} - 6f_{\text{adv}} + 8f_{\text{adv}}\beta_{\text{gas}} + (8 + 4\gamma)f_{\text{adv}}\beta_{\text{mag}} > 0. \quad (16)$$

For the cases without magnetic field, i.e., $\beta_{\text{mag}} = 0$, the above criterion is reduced to (Gu & Lu 2007):

$$2 - 5\beta_{\text{gas}} - 6f_{\text{adv}} + 8\beta_{\text{gas}}f_{\text{adv}} > 0.$$

3. Results

3.1. Stability of disks with different magnetic field strength

We calculate the thermal equilibrium solution of the above equations. The left panel of Fig. 1 shows the value of Δ at $R = 10R_g$ for various \dot{M} and β_{mag} . The value of γ is selected to be 1 as an illustration. We can see that when $\beta_{\text{mag}} \lesssim 0.24$, i.e., the magnetic field is relatively weak, there exist two critical mass accretion rates corresponding to $\Delta = 0$ for each given β_{mag} . When $\beta_{\text{mag}} \gtrsim 0.24$, we always have $\Delta < 0$. That means the disk will be thermally stable for any \dot{M} . The results are similar for the cases of other γ . The right panel of Fig. 1 shows the variation of the two critical accretion rates with β_{mag} , where the solid and dashed lines correspond to the lower and upper critical rates, respectively. The lower critical rate is of interest to us since it is relevant to the thin disks. It corresponds to the threshold of the accretion rate above which the disk is unstable. The upper one corresponds to slim disks with $f_{\text{adv}} \sim 1/3$ (see §3.2). For each γ , the region inside the corresponding parabolic curve denotes the thermally unstable solutions, while the region outside corresponds to the stable ones. The solid lines in the right panel illustrate that the threshold of accretion rate above which the disk is thermally unstable increases with the ratio of magnetic pressure to total pressure. Also, we find that if γ takes a larger value, i.e., the magnetic field reacts more intensely according to a perturbation in the scale height, the critical rate increases more significantly with β_{mag} , and thus a smaller cutoff of the possible β_{mag} for thermal instability appears.

We interpret the above result as follows. Suppose there is a small increase in the temperature T , the radiative cooling will increase following eq. (5), while the response of

viscous heating to temperature is not so simple. As the temperature increases, the radiation pressure and further the scale height increase accordingly, which will result in the increase of viscous heating (ref. eqs. 4 and 8). The key point is that the magnetic field B_φ will decrease (eq. 9), and this effect appears more important when γ is relatively large. Therefore, compared with the case without magnetic field, the increase of the total pressure and thus the scale height will become less significant. In other words, the inclusion of magnetic pressure will result in a weaker dependence of viscous heating on temperature, thus the magnetic field can conspicuously suppress the thermal instability.

3.2. Local thermal equilibria

It is convenient to plot an $\dot{M} \sim \Sigma$ diagram of local disk solutions to directly show the properties of disk stability. For simplicity, we also select the case of $\gamma = 1$ as an instance, which is presented in Figure 2. On those curves in this figure, a negative slop indicates that the solution is thermally unstable. We can see that the threshold of accretion rate for thermal instability, corresponding to the lower turning points of each curve, rises accordingly with increasing Φ_1 , which is in agreement with Fig. 1. For the dotted and the short dashed curves, the thresholds of accretion rate are all $\sim \dot{M}_{\text{Edd}}$, roughly one order of magnitude larger than the case without magnetic field (the long dashed curve), which is supposed to explain the observed high luminosity of *thermally stable* high/soft state of some black hole X-ray binaries (GD04). While for the solid curve, as the corresponding Φ_1 is large enough, the two turning points both disappear.

All the curves in Fig. 2 share the same upper branch. That is because in the cases of large accretion rates, i.e., $\gtrsim 10\dot{M}_{\text{Edd}}$, the advective cooling becomes important and the radiation pressure turns to be very large. Compared with the large radiation pressure, the magnetic pressure can be neglected (the value of $P_{\text{mag}}/P_{\text{rad}}$ is of the order of $\sim 10^{-2}$ at the upper turning point of each curve except the solid one whereas f_{adv} approaches 1/3, and decreases rapidly as the accretion rate increases). Hence, the effect of magnetic field is negligible, and all the curves with different Φ_1 approach the slim disk limit. It is qualitatively the same when γ takes other values. We would like to emphasize that there is no requirement that the disk solution would trace the solution curves in Figure 2 when the increase of the accretion rate is adequately slow. In fact, the conservation of Φ_γ may not hold beyond a thermal timescale.

4. Summary and discussion

Previous analysis shows that the radiation-dominated standard thin disk is thermally unstable when $L \gtrsim 0.06L_{\text{Edd}}$. This conclusion, however, is in conflict with observations of, e.g., the high state of black hole X-ray binaries. Most previous analyses neglect the role of magnetic pressure. In this paper, by taking the magnetic pressure into account, we have revisited the thermal stability of the radiation-dominated standard thin disk. We have derived a general criterion for the thermal instability (eq. 16). We find that the threshold of accretion rate above which the solution becomes unstable increases significantly with the increasing ratio of magnetic pressure and total pressure, β_{mag} (refer to Fig. 1). If β_{mag} is large enough, say $\beta_{\text{mag}} \gtrsim 0.24$ for $\gamma = 1$ or $\beta_{\text{mag}} \gtrsim 0.12$ for $\gamma = 3$, our model predicts that the disk will be stable for any \dot{M} (again refer to Fig. 1). We do not favor this result since the required β_{mag} is significantly larger than the typical value of $\beta_{\text{mag}} \sim 0.1$ given by general numerical simulations. For $\beta_{\text{mag}} \sim 0.1$, from Fig. 1 we expect that the thermal stability of the high state of black hole X-ray binaries with luminosity as high as $0.5L_{\text{Edd}}$ can be explained if $\gamma \gtrsim 3^1$.

The key assumption in our analysis is that during the thermal perturbation the changes of magnetic field and of scale height satisfy a constraint described by eq. (9), namely, *the magnetic field B_ϕ will become weaker with the increase of temperature (or equivalently scale height H)*. The parameter γ in eq. (9) denotes how strong the response of the magnetic field is to a thermal perturbation. Therefore, compared with the case of not including the magnetic pressure, the increase of total pressure and further the scale height and viscous heating with the temperature become weaker (ref. eqs. 2, 4 & 8). This is the reason why the disk tends to be thermally stable.

Unfortunately we are unclear about the value (or the range) of γ because of the complexity of processes such as MRI, dynamo, and reconnection which can strengthen or weaken the magnetic field within timescales shorter than the thermal one. One extreme case is that all these processes are turned off thus the magnetic field is frozen in the accretion flow. In this case, we obviously have $\gamma = 1$, and $\Phi_1 = B_\phi H$ represents half of the toroidal magnetic flux per unit radius. The conservation of Φ_1 is exactly what has been adopted in the thermal stability analysis of a magnetically dominated “low- β ” disk (Machida et al. 2006). In another work studying this type of disk (Oda et al. 2010), the advection rate of toroidal magnetic flux ($\equiv 2v_r B_\phi H$, where $v_r = \dot{M}/2\pi r\Sigma$ is the radial velocity of the accretion flow) is assumed to be constant during a thermal perturbation. This corresponds to $\gamma = 3$, which

¹Throughout the paper we set $\alpha = 0.1$. If the value of α is smaller, the required γ can be smaller to explain the observed stable high state.

is because from eqs. (2), (7) and (8), we can deduce that $v_r \propto H^2$ and therefore $B_\varphi \propto H^{-3}$. We also notice that the model constructed in Oda et al. (2009) corresponds to $\gamma = 2$. In this case, the conservation of $\Phi_2 = B_\varphi H^2$ comes from the specific configuration of both the magnetic field and the outer boundary condition, which does not hold a simple physical meaning. In spite of the uncertainty in the value of γ , we believe that the positive sign of γ , or equivalently the negative response of B_φ to T , should capture the real physics and have essential influence on the thermal stability of radiation-dominated thin disks. It will be interesting to investigate the value of γ by detailed numerical simulations. On the other hand, it is noteworthy that the reversal of toroidal magnetic field was found by vertically stratified shearing box simulations of thin disks (e.g., Brandenburg et al. 1995; Stone et al. 1996), and most recently by global simulations (O’Neill et al. 2010). This phenomenon was reported to occur on ~ 10 orbit timescales, comparable to the thermal timescale. If this phenomena is coupled with the thermal perturbation process, which is unclear to us, it will be a problem for our model, since it will be in conflict with our assumption of the conservation of Φ_γ^2 . Another concern comes when we try to explain the stability of the radiation-dominated simulations presented in Hirose et al. (2009b) based on our model. We find that it requires a rather large value of γ to explain some of their simulations. For example, their model 0519b, the most radiation-dominated one, requires $\gamma \gtrsim 7$. Such a high value feels uncomfortable. Of course, it may be too demanding to require a simple one-dimensional analytical model to completely explain three-dimensional MHD simulations.

Our model predicts that if the magnetic pressure is not too strong, e.g., $\beta_{\text{mag}} \lesssim 0.24$ for $\gamma = 1$, the sources with accretion rates higher than the threshold should still manifest thermal instability. We note in this context that GRS 1915+105 may be one evidence for our prediction. Different from other sources, GRS 1915+105 holds strong variability which seems to be well interpreted as the thermal instability of a radiation-dominated thin disk (e.g., Belloni et al. 1997; Janiuk et al. 2002). GD04 speculated that the reason for the uniqueness of this source might be that it goes to higher luminosity (or equivalently, accretion rate) than all other sources in their sample. Actually GRS1915+105 is likely to be super-Eddington (Done et al. 2004). This prediction could discriminate our model from others such as the one presented in Hirose et al. (2009a).

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²In this context we note that our model actually only requires the negative response of the *absolute value* of B_φ to temperature. But even so, if the field reversal is coupled with thermal perturbation process, since the timescales of the two processes are comparable, our assumption of eq. (9) will still be incorrect.

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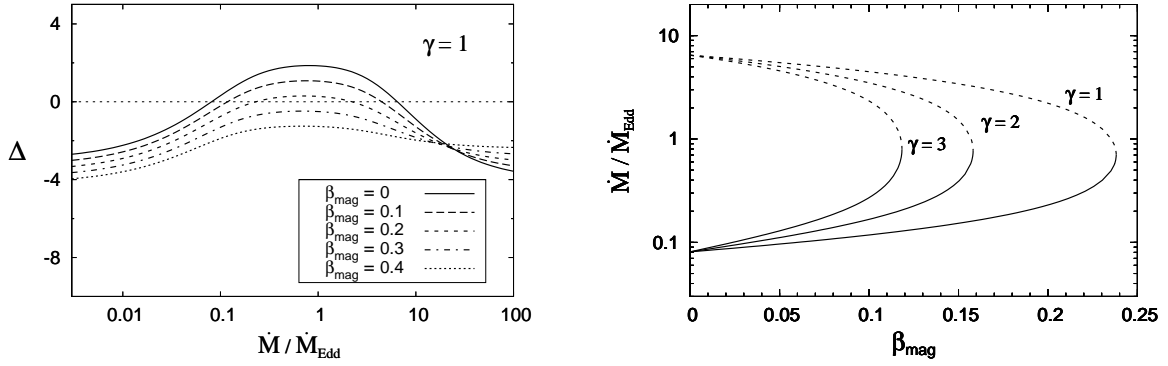


Fig. 1.— The thermal stability of the disk at $R = 10R_g$ for various \dot{M} and β_{mag} . $M = 10M_{\odot}$ and $\alpha = 0.1$ are adopted. The left panel shows the variation of Δ with \dot{M} for given values of β_{mag} , where γ is set to be 1. The solid line corresponds to the none magnetic field cases. The right panel shows the variation of the lower and upper critical accretion rates ($\Delta = 0$) with β_{mag} . Each of the curves corresponds to a specific value for γ .

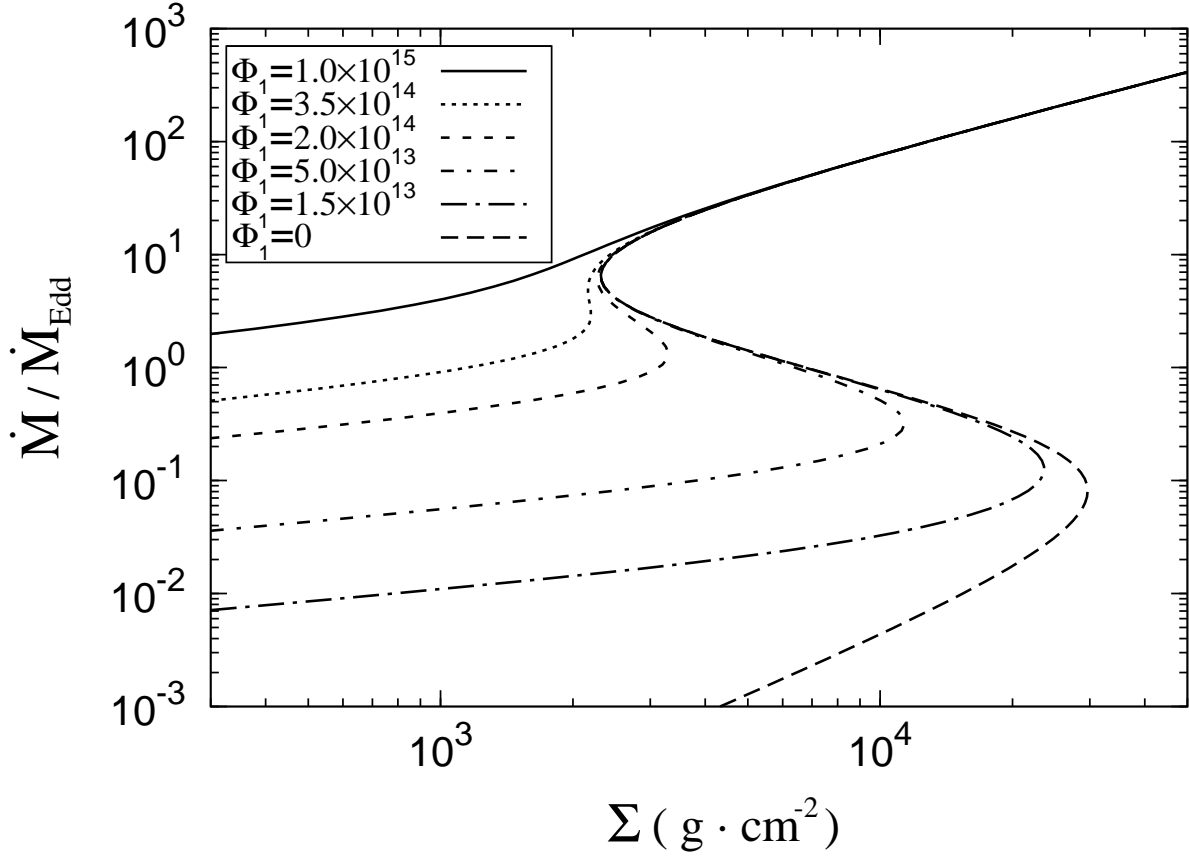


Fig. 2.— The thermal equilibrium curves of the thin disk at $10R_g$ for different Φ_1 , which is defined as $\Phi_1 \equiv B_\varphi H$ and is in unit of $\text{Gs}\cdot\text{cm}$. Other model parameters are $M = 10M_\odot$ and $\alpha = 0.1$.